

EMPIRICAL AND ANALYTICAL STUDY OF INSTABILITIES  
IN HYBRID OPTICAL BISTABLE SYSTEMS

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INTRODUCTION

As it is well known from the work by Gibbs et al<sup>1</sup>, optical turbulence and periodic oscillations are easily seen in hybrid optical bistable devices when a delay is added to the feedback. Such effects, as it was pointed out by Gibbs, may be used to convert cw laser power into a<sub>2</sub> train of light pulses. Furthermore, Okada and Takizawa investigated, theoretically and experimentally, the effect of a delayed feedback on an hybrid electrooptic BOD, with the restriction that the delay time should be less than or comparable to the response time of time of the system. Neyer and Voges<sup>3</sup> demonstrated the neat effect of this feedback delay on the behaviour of electrooptic BOD's, neglecting all the time constants of the system components (photo-diode, amplifier, modulator, etc.).

The main aim of this paper is to determine the characteristics needed by the transmission curve of the system to obtain instabilities in hybrid optical bistable devices.

As it will be shown, it is possible to predict optical instabilities merely from geometrical consideration of the transmission plot. An experimental part is given to show the validity of these predictions.

## FUNDAMENTAL EQUATIONS

The hybrid optical bistable device employed by us is similar to those reported by Garmire<sup>4</sup>, Smith<sup>5</sup> and other workers (Fig. 1).

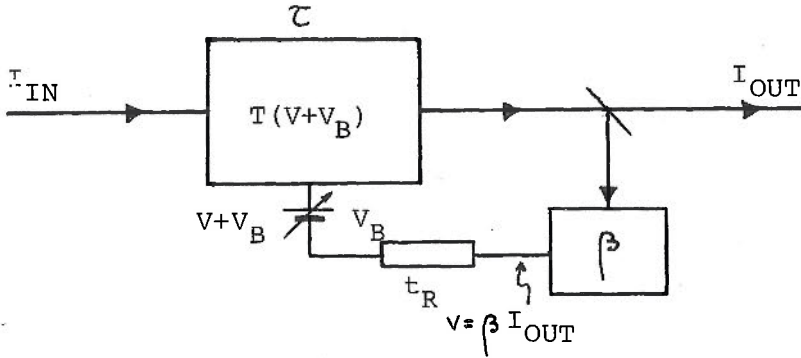


Fig. 1. Hybrid optical bistable system.

Moreover, our theoretical analysis can be applied to any other hybrid system. In any case, it is always verified that

$$I_{OUT}(t) = T(V(t)+V_B) I_{IN} \quad (1)$$

where  $T(V(t)+V_B)$  stands for the transmission characteristics of the electrooptic device and  $V_B$  and  $V(t)$  are the bias and feedback voltages, respectively. If a delay,  $t_R$ , is added, then

$$V(t) + \tau \frac{dV(t)}{dt} = \beta I_{OUT}(t-t_R) \quad (2)$$

From these two equations we can obtain

$$V(t) + \tau \frac{dV(t)}{dt} = \beta \left[ T(V(t-t_R) + V_B) \right] I_{IN} \quad (3)$$

As it is known, the working point for the case of no-delay can be obtained from the intersection of the transmission curve with the straight line described by the equation

$$V = V_B + \beta I_{IN} T(V_B + V) \quad (4)$$

whose slope is given by  $\text{tg}^{-1}(\beta I_{IN})$ . This situation is shown in Fig. 2 where A corresponds to a particular transmission curve and B to equation (4). This working point would correspond to a situation where  $t_R = 0$  (Fig.2).

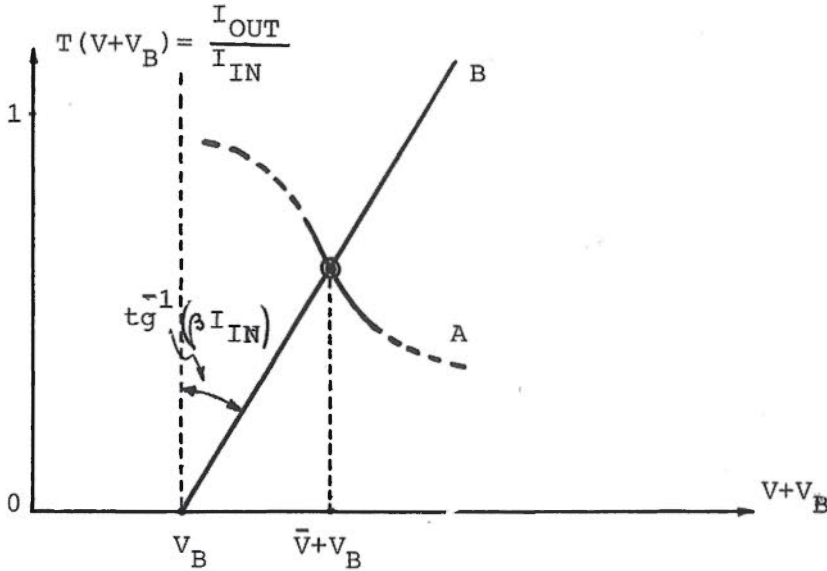


Fig. 2. Equilibrium point.

Let  $V(t)$  be  $\bar{V}$ , the voltage corresponding to the equilibrium point. Hence,  $dV(t)/dt = 0$ . So,

$$\bar{V} = \beta \frac{I_{OUT}}{I_{IN}} = \beta I_{IN} T(\bar{V} + V_B) \quad (5)$$

Making  $u = V - \bar{V}$

$$u + \bar{V} + \tau \frac{d(u + \bar{V})}{dt} = \beta I_{OUT}(t - t_R) = \beta I_{IN} T(V(t - t_R) + V_B) \quad (6)$$

A way to solve this equation is by expansion in Taylor series about the working point. This method is valid for a certain number of conditions whose detailed analysis is beyond the scope of this work. The transmission function will be

$$T(V + V_B) = T(\bar{V} + V_B) + \left( \frac{\partial T}{\partial V} \right)_{\bar{V} + V_B} (V - \bar{V}) + \dots \quad (7)$$

and

$$T(V(t-t_R)+V_B) = T(\bar{V}+V_B) + \left(\frac{dT}{dV}\right)_{\bar{V}+V_B} u(t-t_R) + \dots \quad (8)$$

From (6) and (8)

$$u + \bar{V} + \tau \frac{d(u + \bar{V})}{dt} = \beta I_{IN} \left[ T(\bar{V}+V_B) + \frac{dT}{dV} \bigg|_{\bar{V}+V_B} u(t-t_R) \right] \quad (9)$$

and by using (5)

$$u + \tau \frac{du}{dt} = \beta I_{IN} \left( \frac{dT}{dV} \right)_{\bar{V}+V_B} u(t-t_R) \quad (10)$$

An oscillatory behaviour is described by

$$u(t) = u_0 e^{j\omega t} \quad (11)$$

substituting (11) in (10)

$$1 + j\omega\tau = \beta I_{IN} \left( \frac{dT}{dV} \right)_{\bar{V}+V_B} e^{-j\omega t_R} \quad (12)$$

separating the modulus and argument part of each side,

$$\left| \beta I_{IN} \left( \frac{dT}{dV} \right)_{\bar{V}+V_B} \right| = \sqrt{1 + (\omega\tau)^2} \quad (13)$$

$$\text{tg}(\omega t_R) = -\omega\tau \quad (14)$$

Hence, taking into account that  $\beta I_{IN} \left( \frac{dT}{dV} \right)_{\bar{V}+V_B} < 1$ , at

the equilibrium point, we have

$$\beta I_{IN} \left( \frac{dT}{dV} \right)_{\bar{V}+V_B} \leq -1 \quad (15)$$

and so

$$-\infty \leq \left( \frac{dT}{dV} \right)_{\bar{V}+V_B} < -\frac{1}{\beta I_{IN}} \quad (16)$$

This expression gives the oscillation condition for the  $T(V)$  versus  $V$  plane. It is shown in Fig.3 for a general case.

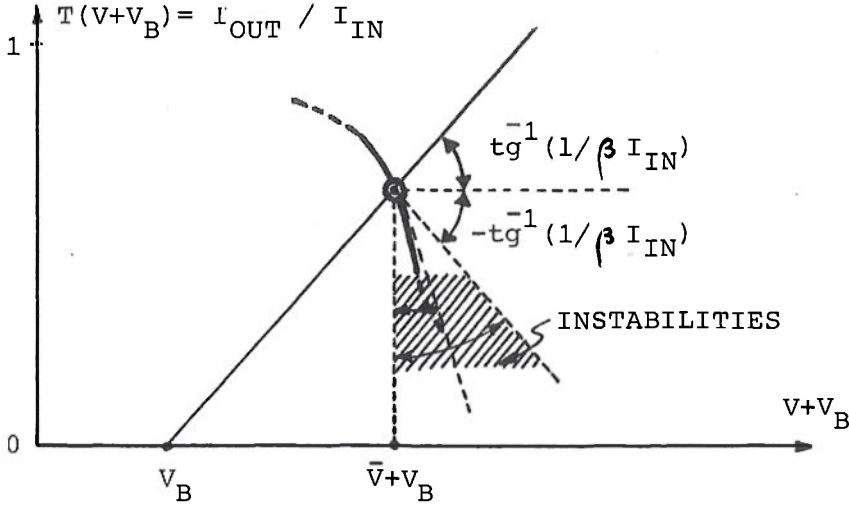


Fig. 3. Geometrical condition for instabilities.

The shadowed region corresponds to the zone where instabilities appear. If the slope of the transmission curve is inside this region, the system will oscillate.

The above condition can be extended for  $I_{OUT}$  vs.  $I_{IN}$  and  $\beta I_{OUT}$  vs.  $V_B$  planes. In the first case,

$$\frac{dI_{OUT}}{dI_{IN}} = \frac{T}{1 - \beta I_{IN} \left( \frac{dT}{dV} \right)} \quad (17)$$

From this equation, expression (16) can be written

$$0 \leq \frac{dI_{OUT}}{dI_{IN}} \leq \frac{T}{2} \quad (18)$$

where  $T = I_{OUT} / I_{IN}$ . This inequality is shown in Fig.4 where instabilities appear for

$$0 \leq \text{tg } \theta_2 \leq \frac{1}{2} \text{tg } \theta_1 \quad (19)$$

with  $\operatorname{tg} \theta_1 = I_{\text{OUT}} / I_{\text{IN}} = T$  and  $\operatorname{tg} \theta_2 = d(I_{\text{OUT}}) / d(I_{\text{IN}})$

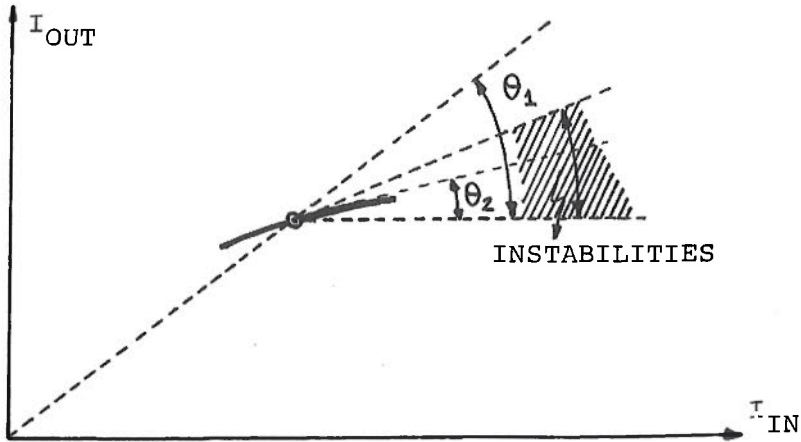


Fig. 4. Instabilities for the  $I_{\text{OUT}}$  vs.  $I_{\text{IN}}$  plane.

Similarly, at the  $\beta I_{\text{OUT}}$  vs.  $V_B$  plane

$$\frac{d I_{\text{OUT}}}{d V_B} = \frac{I_{\text{IN}} \left( \frac{dT}{dV} \right)}{1 - \beta I_{\text{IN}} \left( \frac{dT}{dV} \right)} \quad (20)$$

and with (16)

$$-1 \leq \frac{d(\beta I_{\text{OUT}})}{dV_B} \leq -\frac{1}{2} \quad (21)$$

This situation is depicted in Fig. 5.

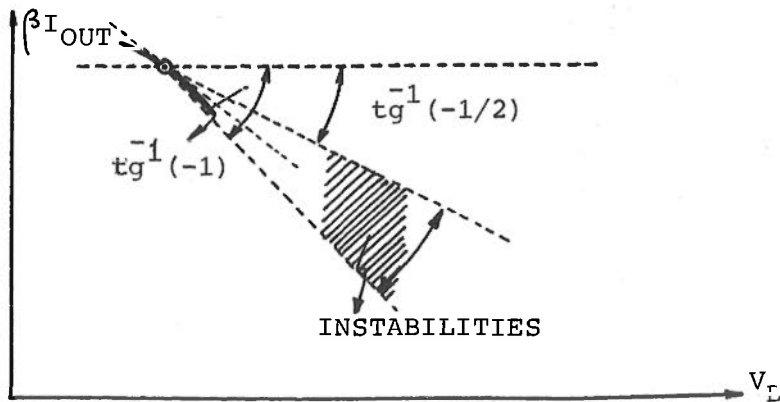


Fig. 5. Instabilities for the  $\beta I_{\text{OUT}}$  vs.  $V_B$  plane.

## EXPERIMENTAL VERIFICATION

The validity of the above model has been tested in two ways. The first one was to apply our theory to the empirical results previously reported by Gibbs et al.<sup>1</sup> and Okada et al.<sup>2</sup>. The regions where their instabilities appear are in total agreement with our theory.

Moreover, to study inequality (21) the experimental setup of Fig. 1 was used. A very low frequency ( $\sim 1$  Hz) triangular voltage bias was added to the feedback instead of the usual constant one. A twisted nematic liquid crystal cell was employed as non-linear element. Further experimental details are given elsewhere<sup>4</sup>. The laser beam impinges orthogonally with respect to the cell.

A periodic oscillation, 20 Hz of frequency, was found at the regions where instabilities had been predicted by our theory.

The instabilities region corresponds to the shadowed zone in Fig. 6, where transmission vs. voltage applied to the cell is represented.

We can conclude that empirical results are in excellent agreement with the predictions of our theory.

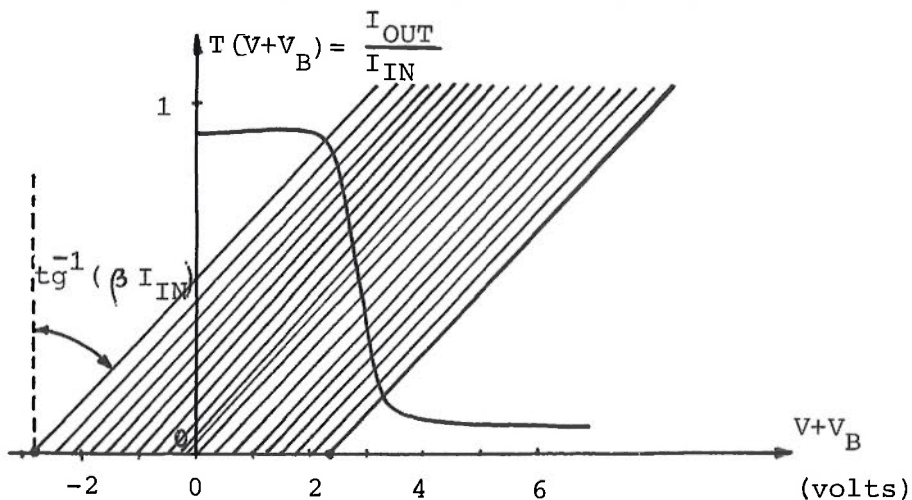


Fig. 6. Experimental region for oscillation.

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